5

Axial Flow Compressors and Fans

5.1 INTRODUCTION

As mentioned in Chapter 4, the maximum pressure ratio achieved in centrifugal compressors is about 4:1 for simple machines (unless multi-staging is used) at an efficiency of about 70–80%. The axial flow compressor, however, can achieve higher pressures at a higher level of efficiency. There are two important characteristics of the axial flow compressor—high-pressure ratios at good efficiency and thrust per unit frontal area. Although in overall appearance, axial turbines are very similar, examination of the blade cross-section will indicate a big difference. In the turbine, inlet passage area is greater than the outlet. The opposite occurs in the compressor, as shown in Fig. 5.1.

Thus the process in turbine blades can be described as an accelerating flow, the increase in velocity being achieved by the nozzle. However, in the axial flow compressor, the flow is decelerating or diffusing and the pressure rise occurs when the fluid passes through the blades. As mentioned in the chapter on diffuser design (Chapter 4, Sec. 4.7), it is much more difficult to carry out efficient diffusion due to the breakaway of air molecules from the walls of the diverging passage. The air molecules that break away tend to reverse direction and flow back in the direction of the pressure gradient. If the divergence is too rapid, this may result in the formation of eddies and reduction in useful pressure rise. During acceleration in a nozzle, there is a natural tendency for the air to fill the passage

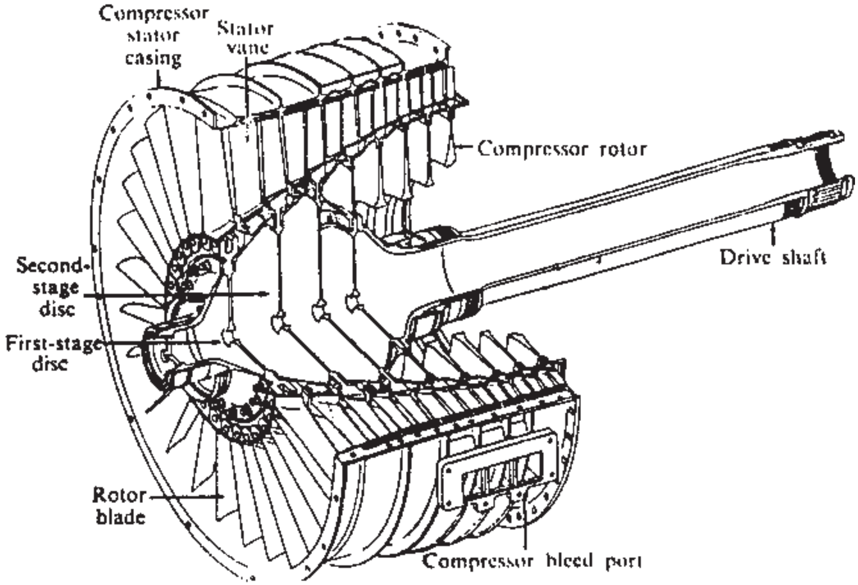

Figure 5.1 Cutaway sketch of a typical axial compressor assembly: the General Electric J85 compressor. (Courtesy of General Electric Co.)

walls closely (only the normal friction loss will be considered in this case). Typical blade sections are shown in Fig. 5.2. Modern axial flow compressors may give efficiencies of 86–90%—compressor design technology is a well-developed field. Axial flow compressors consist of a number of stages, each stage being formed by a stationary row and a rotating row of blades.

Figure 5.3 shows how a few compressor stages are built into the axial compressor. The rotating blades impart kinetic energy to the air while increasing air pressure and the stationary row of blades redirect the air in the proper direction and convert a part of the kinetic energy into pressure. The flow of air through the compressor is in the direction of the axis of the compressor and, therefore, it is called an axial flow compressor. The height of the blades is seen to decrease as the fluid moves through the compressor. As the pressure increases in the direction of flow, the volume of air decreases. To keep the air velocity the same for each stage, the blade height is decreased along the axis of the compressor. An extra row of fixed blades, called the inlet guide vanes, is fitted to the compressor inlet. These are provided to guide the air at the correct angle onto the first row of moving blades. In the analysis of the highly efficient axial flow compressor, the 2-D flow through the stage is very important due to cylindrical symmetry.

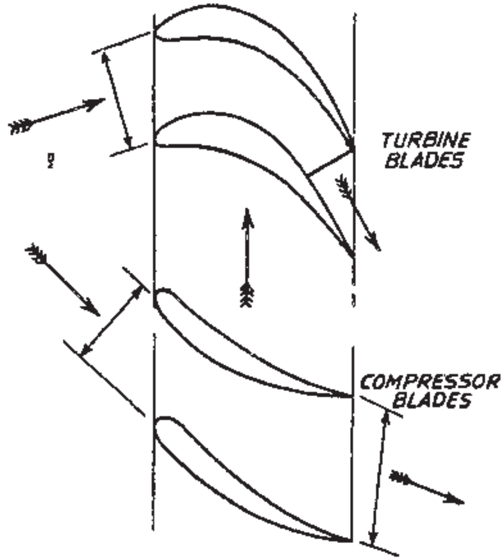

Figure 5.2 Compressor and turbine blade passages: turbine and compressor housing.

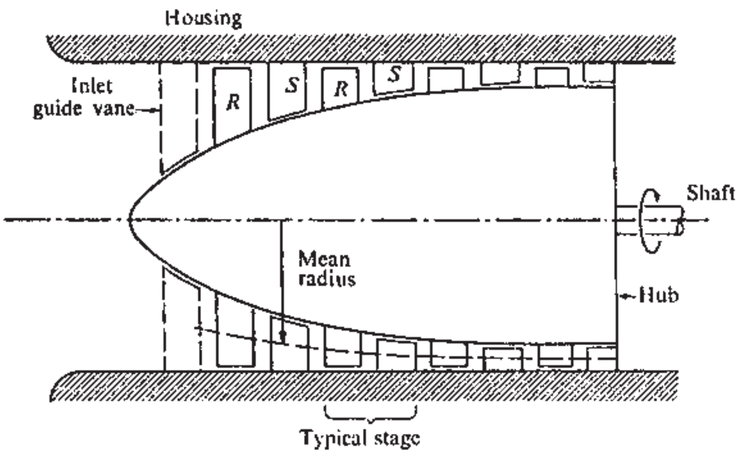

Figure 5.3 Schematic of an axial compressor section.

The flow is assumed to take place at a mean blade height, where the blade peripheral velocities at the inlet and outlet are the same. No flow is assumed in the radial direction.

5.2 VELOCITY DIAGRAM

The basic principle of axial compressor operation is that kinetic energy is imparted to the air in the rotating blade row, and then diffused through passages of both rotating and stationary blades. The process is carried out over multiple numbers of stages. As mentioned earlier, diffusion is a deceleration process. It is efficient only when the pressure rise per stage is very small. The blading diagram and the velocity triangle for an axial flow compressor stage are shown in Fig. 5.4.

Air enters the rotor blade with absolute velocity C_1 at an angle α_1 measured from the axial direction. Air leaves the rotor blade with absolute velocity C_2 at an angle α_2. Air passes through the diverging passages formed between the rotor blades. As work is done on the air in the rotor blades, C_2 is larger than C_1. The rotor row has tangential velocity U. Combining the two velocity vectors gives the relative velocity at inlet V_1 at an angle β_1. V_2 is the relative velocity at the rotor outlet. It is less than V_1, showing diffusion of the relative velocity has taken place with some static pressure rise across the rotor blades. Turning of the air towards the axial direction is brought about by the camber of the blades. Euler's equation

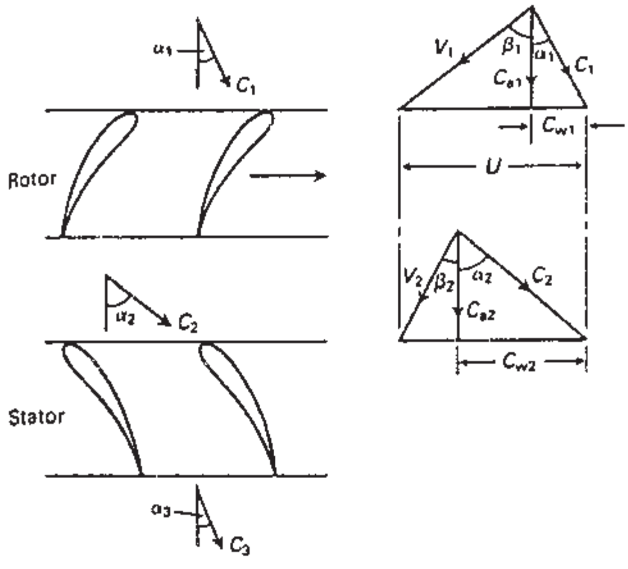

Figure 5.4 Velocity diagrams for a compressor stage.

provides the work done on the air:

$$W_c = U(C_{w2} - C_{w1}) \tag{5.1}$$

Using the velocity triangles, the following basic equations can be written:

$$\frac{U}{C_a} = \tan\alpha_1 + \tan\beta_1 \tag{5.2}$$

$$\frac{U}{C_a} = \tan\alpha_2 + \tan\beta_2 \tag{5.3}$$

in which $C_a = C_{a1} = C_2$ is the axial velocity, assumed constant through the stage. The work done equation [Eq. (5.1)] may be written in terms of air angles:

$$W_c = UC_a(\tan\alpha_2 - \tan\alpha_1) \tag{5.4}$$

also,

$$W_c = UC_a(\tan\beta_1 - \tan\beta_2) \tag{5.5}$$

The whole of this input energy will be absorbed usefully in raising the pressure and velocity of the air and for overcoming various frictional losses. Regardless of the losses, all the energy is used to increase the stagnation temperature of the air, $\triangle T_{0s}$. If the velocity of air leaving the first stage C_3 is made equal to C_1, then the stagnation temperature rise will be equal to the static temperature rise, $\triangle T_s$. Hence:

$$T_{0s} = \Delta T_s = \frac{UC_a}{C_p}(\tan\beta_1 - \tan\beta_2) \tag{5.6}$$

Equation (5.6) is the theoretical temperature rise of the air in one stage. In reality, the stage temperature rise will be less than this value due to 3-D effects in the compressor annulus. To find the actual temperature rise of the air, a factor λ, which is between 0 and 100%, will be used. Thus the actual temperature rise of the air is given by:

$$T_{0s} = \frac{\lambda UC_a}{C_p}(\tan\beta_1 - \tan\beta_2) \tag{5.7}$$

If R_s is the stage pressure ratio and η_s is the stage isentropic efficiency, then:

$$R_s = \left[1 + \frac{\eta_s \Delta T_{0s}}{T_{01}}\right]^{\gamma/(\gamma-1)} \tag{5.8}$$

where T_{01} is the inlet stagnation temperature.

5.3 DEGREE OF REACTION

The degree of reaction, Λ, is defined as:

$$\Lambda = \frac{\text{Static enthalpy rise in the rotor}}{\text{Static enthalpy rise in the whole stage}} \tag{5.9}$$

The degree of reaction indicates the distribution of the total pressure rise into the two types of blades. The choice of a particular degree of reaction is important in that it affects the velocity triangles, the fluid friction and other losses.

Let:

ΔT_{A} = the static temperature rise in the rotor

ΔT_{B} = the static temperature rise in the stator

Using the work input equation [Eq. (5.4)], we get:

$$\begin{aligned} W_{\mathrm{c}} &= C_{\mathrm{p}}(\Delta T_{\mathrm{A}} + \Delta T_{\mathrm{B}}) = \Delta T_{\mathrm{S}} \\ &\left.\begin{aligned} &= UC_{\mathrm{a}}(\tan\beta_1 - \tan\beta_2) \\ &= UC_{\mathrm{a}}(\tan\alpha_2 - \tan\alpha_1) \end{aligned}\right\} \end{aligned} \tag{5.10}$$

But since all the energy is transferred to the air in the rotor, using the steady flow energy equation, we have:

$$W_{\mathrm{c}} = C_{\mathrm{p}}\Delta T_{\mathrm{A}} + \frac{1}{2}(C_2^2 - C_1^2) \tag{5.11}$$

Combining Eqs. (5.10) and (5.11), we get:

$$C_{\mathrm{p}}\Delta T_{\mathrm{A}} = UC_{\mathrm{a}}(\tan\alpha_2 - \tan\alpha_1) - \frac{1}{2}(C_2^2 - C_1^2)$$

from the velocity triangles,

$$C_2 = C_{\mathrm{a}}\cos\alpha_2 \quad \text{and} \quad C_1 = C_{\mathrm{a}}\cos\alpha_1$$

Therefore,

$$\begin{aligned} C_{\mathrm{p}}\Delta T_{\mathrm{A}} &= UC_{\mathrm{a}}(\tan\alpha_2 - \tan\alpha_1) - \tfrac{1}{2}C_{\mathrm{a}}^2(\sec^2\alpha_2 - \sec^2\alpha_1) \\ &= UC_{a}(\tan\alpha_2 - \tan\alpha_1) - \tfrac{1}{2}C_{\mathrm{a}}^2(\tan^2\alpha_2 - \tan^2\alpha_1) \end{aligned}$$

Using the definition of degree of reaction,

$$\begin{aligned} \Lambda &= \frac{\Delta T_{\mathrm{A}}}{\Delta T_{\mathrm{A}} + \Delta T_{\mathrm{B}}} \\ &= \frac{UC_{\mathrm{a}}(\tan\alpha_2 - \tan\alpha_1) - \frac{1}{2}C_{\mathrm{a}}^2(\tan^2\alpha_2 - \tan^2\alpha_1)}{UC_{\mathrm{a}}(\tan\alpha_2 - \tan\alpha_1)} \\ &= 1 - \tfrac{C_{\mathrm{a}}}{U}(\tan\alpha_2 + \tan\alpha_1) \end{aligned}$$

But from the velocity triangles, adding Eqs. (5.2) and (5.3),

$$\frac{2U}{C_a} = (\tan\alpha_1 + \tan\beta_1 + \tan\alpha_2 + \tan\beta_2)$$

Therefore,

$$\Lambda = \frac{C_a}{2U}\left(\frac{2U}{C_a} - \frac{2U}{C_a} + \tan\beta_1 + \tan\beta_2\right)$$

$$= \frac{C_a}{2U}(\tan\beta_1 + \tan\beta_2) \quad (5.12)$$

Usually the degree of reaction is set equal to 50%, which leads to this interesting result:

$$(\tan\beta_1 + \tan\beta_2) = \frac{U}{C_a}.$$

Again using Eqs. (5.1) and (5.2),

$$\tan\alpha_1 = \tan\beta_2, \quad \text{i.e.,} \quad \alpha_1 = \beta_2$$

$$\tan\beta_1 = \tan\alpha_2, \quad \text{i.e.,} \quad \alpha_2 = \beta_1$$

As we have assumed that C_a is constant through the stage,

$$C_a = C_1\cos\alpha_1 = C_3\cos\alpha_3.$$

Since we know $C_1 = C_3$, it follows that $\alpha_1 = \alpha_3$. Because the angles are equal, $\alpha_1 = \beta_2 = \alpha_3$, and $\beta_1 = \alpha_2$. Under these conditions, the velocity triangles become symmetric. In Eq. (5.12), the ratio of axial velocity to blade velocity is called the flow coefficient and denoted by Φ. For a reaction ratio of 50%, $(h_2 - h_1) = (h_3 - h_1)$, which implies the static enthalpy and the temperature increase in the rotor and stator are equal. If for a given value of C_a/U, β_2 is chosen to be greater than α_2 (Fig. 5.5), then the static pressure rise in the rotor is greater than the static pressure rise in the stator and the reaction is greater than 50%.

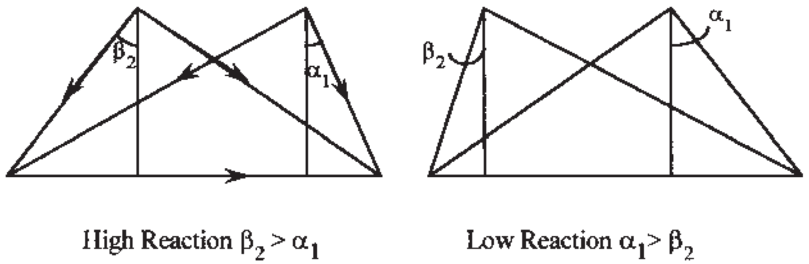

Figure 5.5 Stage reaction.

Conversely, if the designer chooses β_2 less than β_1, the stator pressure rise will be greater and the reaction is less than 50%.

5.4 STAGE LOADING

The stage-loading factor Ψ is defined as:

$$\begin{aligned}\Psi &= \frac{W_c}{mU^2} = \frac{h_{03} - h_{01}}{U^2} \\ &= \frac{\lambda(C_{w2} - C_{w1})}{U} \\ &= \frac{\lambda C_a}{U}(\tan\alpha_2 - \tan\alpha_1) \\ \Psi &= \lambda\Phi\,(\tan\alpha_2 - \tan\alpha_1)\end{aligned} \tag{5.13}$$

5.5 LIFT-AND-DRAG COEFFICIENTS

The stage-loading factor Ψ may be expressed in terms of the lift-and-drag coefficients. Consider a rotor blade as shown in Fig. 5.6, with relative velocity vectors V_1 and V_2 at angles β_1 and β_2. Let $\tan(\beta_m) = (\tan(\beta_1) + \tan(\beta_2))/2$. The flow on the rotor blade is similar to flow over an airfoil, so lift-and-drag forces will be set up on the blade while the forces on the air will act on the opposite direction.

The tangential force on each moving blade is:

$$F_x = L\cos\beta_m + D\sin\beta_m$$

$$F_x = L\cos\beta_m\left[1 + \left(\frac{C_D}{C_L}\right)\tan\beta_m\right] \tag{5.14}$$

where: L = lift and D = drag.

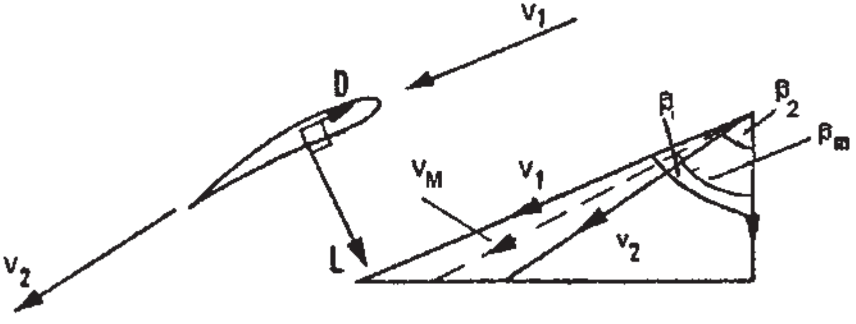

Figure 5.6 Lift-and-drag forces on a compressor rotor blade.

The lift coefficient is defined as:

$$C_L = \frac{L}{0.5\rho V_m^2 A} \tag{5.15}$$

where the blade area is the product of the chord c and the span l.

Substituting $V_m = \frac{C_a}{\cos\beta_m}$ into the above equation,

$$F_x = \frac{\rho C_a^2 c l C_L}{2} \sec\beta_m \left[1 + \left(\frac{C_D}{C_L}\right)\tan\beta_m\right] \tag{5.16}$$

The power delivered to the air is given by:

$$\begin{aligned} UF_x &= m(h_{03} - h_{01}) \\ &= \rho C_a l s(h_{03} - h_{01}) \end{aligned} \tag{5.17}$$

considering the flow through one blade passage of width s.

Therefore,

$$\begin{aligned} &= \frac{h_{03} - h_{01}}{U^2} \\ &= \frac{F_x}{\rho C_a l s U} \\ &= \frac{1}{2}\left(\frac{C_a}{U}\right)\left(\frac{c}{s}\right)\sec\beta_m(C_L + C_D\tan\beta_m) \\ &= \frac{1}{2}\left(\frac{c}{s}\right)\sec\beta_m(C_L + C_D\tan\beta_m) \end{aligned} \tag{5.18}$$

For a stage in which $\beta_m = 45°$, efficiency will be maximum. Substituting this back into Eq. (5.18), the optimal blade-loading factor is given by:

$$\Psi_{opt} = \frac{\varphi}{\sqrt{2}}\left(\frac{c}{s}\right)(C_L + C_D) \tag{5.19}$$

For a well-designed blade, C_D is much smaller than C_L, and therefore the optimal blade-loading factor is approximated by:

$$\Psi_{opt} = \frac{\varphi}{\sqrt{2}}\left(\frac{c}{s}\right)C_L \tag{5.20}$$

5.6 CASCADE NOMENCLATURE AND TERMINOLOGY

Studying the 2-D flow through cascades of airfoils facilitates designing highly efficient axial flow compressors. A cascade is a row of geometrically similar blades arranged at equal distance from each other and aligned to the flow direction. Figure 5.7, which is reproduced from Howell's early paper on cascade theory and performance, shows the standard nomenclature relating to airfoils in cascade.

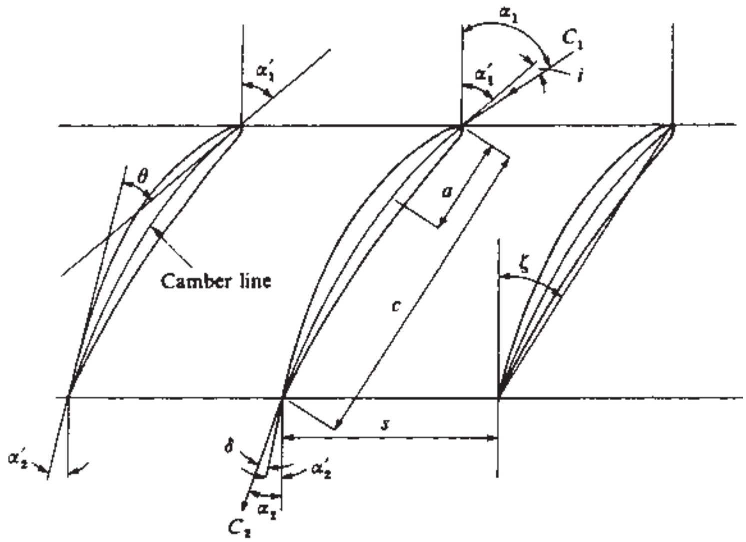

Figure 5.7 Cascade nomenclature.

$\alpha_1{}'$ and $\alpha_2{}'$ are the camber angles of the entry and exit tangents the camber line makes with the axial direction. The blade camber angle $\theta = \alpha_1' - \alpha_2'$. The chord c is the length of the perpendicular of the blade profile onto the chord line. It is approximately equal to the linear distance between the leading edge and the trailing edge. The stagger angle ξ is the angle between the chord line and the axial direction and represents the angle at which the blade is set in the cascade. The pitch s is the distance in the direction of rotation between corresponding points on adjacent blades. The incidence angle i is the difference between the air inlet angle (α_1) and the blade inlet angle $\left(\alpha_1'\right)$. That is, $i = \alpha_1 - \alpha_1'$. The deviation angle (δ) is the difference between the air outlet angle (α_2) and the blade outlet angle $\left(\alpha_2'\right)$. The air deflection angle, $\varepsilon = \alpha_1 - \alpha_2$, is the difference between the entry and exit air angles.

A cross-section of three blades forming part of a typical cascade is shown in Fig. 5.7. For any particular test, the blade camber angle θ, its chord c, and the pitch (or space) s will be fixed and the blade inlet and outlet angles α_1' and α_2' are determined by the chosen setting or stagger angle ξ. The angle of incidence, i, is then fixed by the choice of a suitable air inlet angle α_1, since $i = \alpha_1 - \alpha_1'$. An appropriate setting of the turntable on which the cascade is mounted can accomplish this. With the cascade in this position the pressure and direction measuring instruments are then traversed along the blade row in the upstream and downstream position. The results of the traverses are usually presented as shown

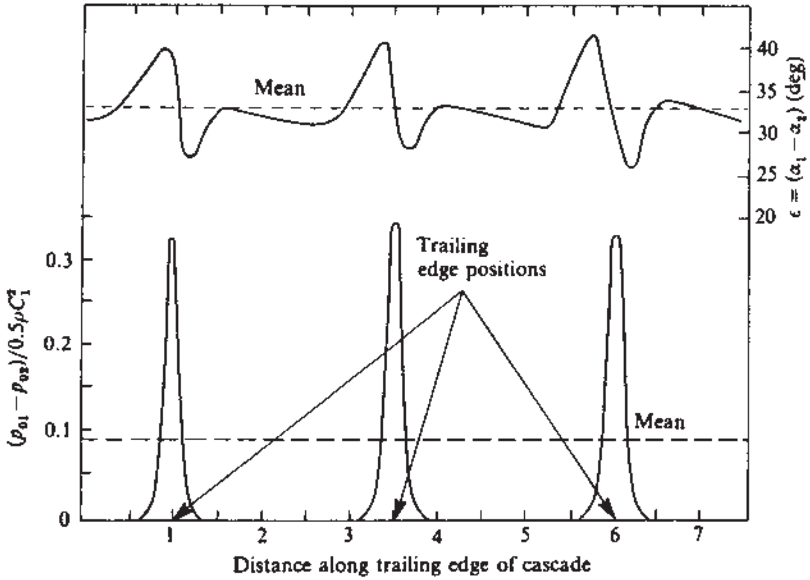

Figure 5.8 Variation of stagnation pressure loss and deflection for cascade at fixed incidence.

in Fig. 5.8. The stagnation pressure loss is plotted as a dimensionless number given by:

$$\text{Stagnation pressure loss coefficient} = \frac{P_{01} - P_{02}}{0.5\rho C_1^2} \tag{5.21}$$

This shows the variation of loss of stagnation pressure and the air deflection, $\varepsilon = \alpha_1 - \alpha_2$, covering two blades at the center of the cascade. The curves of Fig. 5.8 can now be repeated for different values of incidence angle, and the whole set of results condensed to the form shown in Fig. 5.9, in which the mean loss and mean deflection are plotted against incidence for a cascade of fixed geometrical form.

The total pressure loss owing to the increase in deflection angle of air is marked when i is increased beyond a particular value. The stalling incidence of the cascade is the angle at which the total pressure loss is twice the minimum cascade pressure loss. Reducing the incidence i generates a negative angle of incidence at which stalling will occur.

Knowing the limits for air deflection without very high (more than twice the minimum) total pressure loss is very useful for designers in the design of efficient compressors. Howell has defined nominal conditions of deflection for

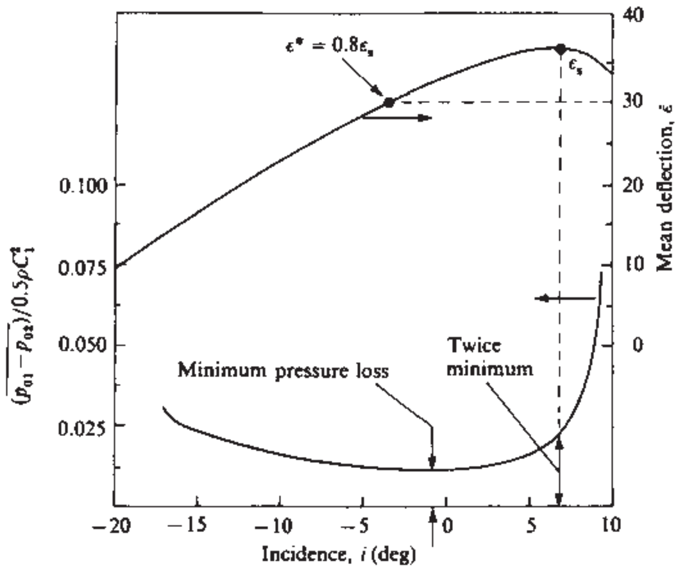

Figure 5.9 Cascade mean deflection and pressure loss curves.

a cascade as 80% of its stalling deflection, that is:

$$\varepsilon^* = 0.8\varepsilon_s \tag{5.22}$$

where ε_s is the stalling deflection and ε^* is the nominal deflection for the cascade.

Howell and Constant also introduced a relation correlating nominal deviation δ^* with pitch chord ratio and the camber of the blade. The relation is given by:

$$\delta^* = m\theta\left(\frac{s}{l}\right)^n \tag{5.23}$$

For compressor cascade, $n = \frac{1}{2}$, and for the inlet guide vane in front of the compressor, $n = 1$. Hence, for a compressor cascade, nominal deviation is given by:

$$\delta^* = m\theta\left(\frac{s}{l}\right)^{\frac{1}{2}} \tag{5.24}$$

The approximate value suggested by Constant is 0.26, and Howell suggested a modified value for m:

$$m = 0.23\left(\frac{2a}{l}\right)^2 + 0.1\left(\frac{\alpha_2^*}{50}\right) \tag{5.25}$$

where the maximum camber of the cascade airfoil is at a distance a from the leading edge and α_2^* is the nominal air outlet angle.

Then,

$$\alpha_2^* = \beta_2 + \delta^*$$

$$= \beta_2 + m\theta\left(\frac{s}{l}\right)^{\frac{1}{2}}$$

and,

$$\alpha_1^* - \alpha_2^* = \varepsilon^*$$

or:

$$\alpha_1^* = \alpha_2^* + \varepsilon^*$$

Also,

$$i^* = \alpha_1^* - \beta_1 = \alpha_2^* + \varepsilon^* - \beta_1$$

5.7 3-D CONSIDERATION

So far, all the above discussions were based on the velocity triangle at one particular radius of the blading. Actually, there is a considerable difference in the velocity diagram between the blade hub and tip sections, as shown in Fig. 5.10.

The shape of the velocity triangle will influence the blade geometry, and, therefore, it is important to consider this in the design. In the case of a compressor with high hub/tip ratio, there is little variation in blade speed from root to tip. The shape of the velocity diagram does not change much and, therefore, little variation in pressure occurs along the length of the blade. The blading is of the same section at all radii and the performance of the compressor stage is calculated from the performance of the blading at the mean radial section. The flow along the compressor is considered to be 2-D. That is, in 2-D flow only whirl and axial flow velocities exist with no radial velocity component. In an axial flow compressor in which high hub/tip radius ratio exists on the order of 0.8, 2-D flow in the compressor annulus is a fairly reasonable assumption. For hub/tip ratios lower than 0.8, the assumption of two-dimensional flow is no longer valid. Such compressors, having long blades relative to the mean diameter, have been used in aircraft applications in which a high mass flow requires a large annulus area but a small blade tip must be used to keep down the frontal area. Whenever the fluid has an angular velocity as well as velocity in the direction parallel to the axis of rotation, it is said to have "vorticity." The flow through an axial compressor is vortex flow in nature. The rotating fluid is subjected to a centrifugal force and to balance this force, a radial pressure gradient is necessary. Let us consider the pressure forces on a fluid element as shown in Fig. 5.10. Now, resolve

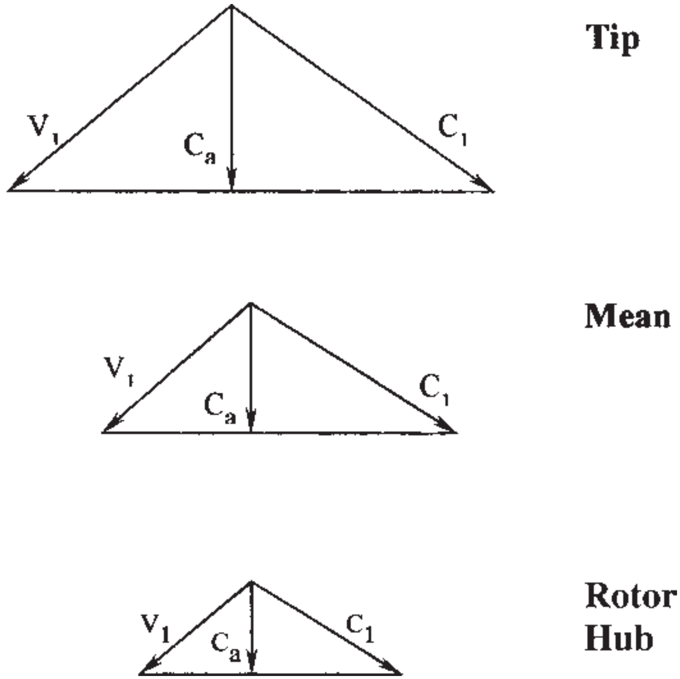

Figure 5.10 Variation of velocity diagram along blade.

the forces in the radial direction Fig. 5.11:

$$d\theta (P + dP)(r + dr) - Pr\, d\theta - 2\left(P + \frac{dP}{2}\right) dr \frac{d\theta}{2}$$
$$= \rho\, dr\, r\, d\theta \frac{C_w^2}{r} \qquad (5.26)$$

or

$$(P + dP)(r + dr) - Pr - \left(P + \frac{dP}{2}\right) dr = \rho\, dr\, C_w^2$$

where: P is the pressure, ρ, the density, C_w, the whirl velocity, r, the radius.

After simplification, we get the following expression:

$$Pr + P\, dr + r\, dP + dP\, dr - Pr + \rho\, dr - \frac{1}{2} dP\, dr = \rho\, dr\, C_w^2$$

or:

$$r\, dP = \rho\, dr\, C_w^2$$

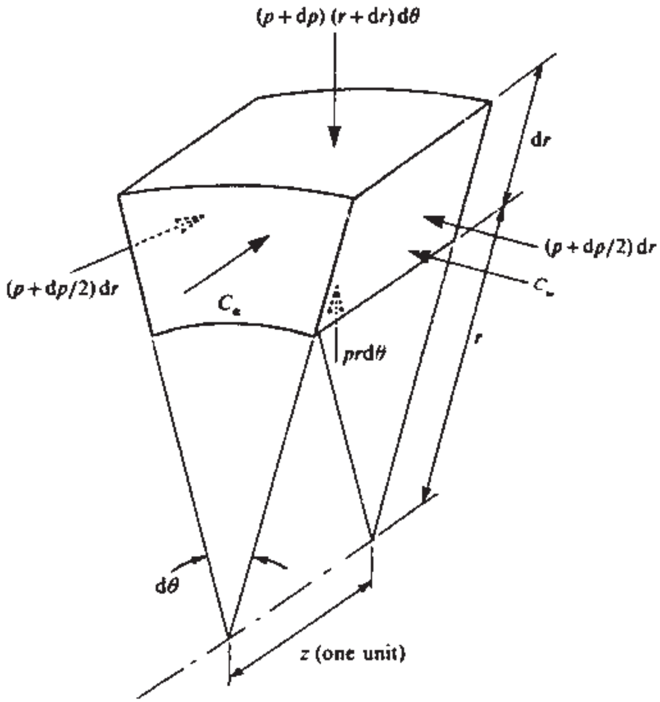

Figure 5.11 Pressure forces on a fluid element.

That is,

$$\frac{1}{\rho}\frac{\mathrm{d}P}{\mathrm{d}r} = \frac{C_{\mathrm{w}}^2}{r} \tag{5.27}$$

The approximation represented by Eq. (5.27) has become known as radial equilibrium.

The stagnation enthalpy h_0 at any radius r where the absolute velocity is C may be rewritten as:

$$h_0 = h + \frac{1}{2}C_{\mathrm{a}}^2 + \frac{1}{2}C_{\mathrm{w}}^2; \left(h = c_{\mathrm{p}}T, \quad \text{and} \quad C^2 = C_{\mathrm{a}}^2 + C_{\mathrm{w}}^2\right)$$

Differentiating the above equation w.r.t. r and equating it to zero yields:

$$\frac{\mathrm{d}h_0}{\mathrm{d}r} = \frac{\gamma}{\gamma - 1} \times \frac{1}{\rho}\frac{\mathrm{d}P}{\mathrm{d}r} + \frac{1}{2}\left(0 + 2C_{\mathrm{w}}\frac{\mathrm{d}C_{\mathrm{w}}}{\mathrm{d}r}\right)$$

or:

$$\frac{\gamma}{\gamma-1} \times \frac{1}{\rho}\frac{\mathrm{d}P}{\mathrm{d}r} + C_{\mathrm{w}}\frac{\mathrm{d}C_{\mathrm{w}}}{\mathrm{d}r} = 0$$

Combining this with Eq. (5.27):

$$\frac{\gamma}{\gamma-1}\frac{C_{\mathrm{w}}^2}{r} + C_{\mathrm{w}}\frac{\mathrm{d}C_{\mathrm{w}}}{\mathrm{d}r} = 0$$

or:

$$\frac{\mathrm{d}C_{\mathrm{w}}}{\mathrm{d}r} = -\frac{\gamma}{\gamma-1}\frac{C_{\mathrm{w}}}{r}$$

Separating the variables,

$$\frac{\mathrm{d}C_{\mathrm{w}}}{C_{\mathrm{w}}} = -\frac{\gamma}{\gamma-1}\frac{\mathrm{d}r}{r}$$

Integrating the above equation

$$\int\frac{\mathrm{d}C_{\mathrm{w}}}{C_{\mathrm{w}}} = -\frac{\gamma}{\gamma-1}\int\frac{\mathrm{d}r}{r}$$

$$-\frac{\gamma}{\gamma-1}\ \ln C_{\mathrm{w}}r = c \quad \text{where } c \text{ is a constant.}$$

Taking antilog on both sides,

$$\frac{\gamma}{\gamma-1} \times C_{\mathrm{w}} \times r = e^{\mathrm{c}}$$

Therefore, we have

$$C_{\mathrm{w}}r = \text{constant} \tag{5.28}$$

Equation (5.28) indicates that the whirl velocity component of the flow varies inversely with the radius. This is commonly known as free vortex. The outlet blade angles would therefore be calculated using the free vortex distribution.

5.8 MULTI-STAGE PERFORMANCE

An axial flow compressor consists of a number of stages. If R is the overall pressure ratio, R_{s} is the stage pressure ratio, and N is the number of stages, then the total pressure ratio is given by:

$$R = (R_{\mathrm{s}})^N \tag{5.29}$$

Equation (5.29) gives only a rough value of R because as the air passes through the compressor the temperature rises continuously. The equation used to find stage pressure is given by:

$$R_s = \left[1 + \frac{\eta_s \Delta T_{0s}}{T_{01}}\right]^{\frac{\gamma}{\gamma-1}} \tag{5.30}$$

The above equation indicates that the stage pressure ratio depends only on inlet stagnation temperature T_{01}, which goes on increasing in the successive stages. To find the value of R, the concept of polytropic or small stage efficiency is very useful. The polytropic or small stage efficiency of a compressor is given by:

$$\eta_{\infty,c} = \left(\frac{\gamma - 1}{\gamma}\right)\left(\frac{n}{n-1}\right)$$

or:

$$\left(\frac{n}{n-1}\right) = \eta_s \left(\frac{\gamma}{\gamma - 1}\right)$$

where $\eta_s = \eta_{\infty,c}$ = small stage efficiency.

The overall pressure ratio is given by:

$$R = \left[1 + \frac{N \Delta T_{0s}}{T_{01}}\right]^{\frac{n}{n-1}} \tag{5.31}$$

Although Eq. (5.31) is used to find the overall pressure ratio of a compressor, in actual practice the step-by-step method is used.

5.9 AXIAL FLOW COMPRESSOR CHARACTERISTICS

The forms of characteristic curves of axial flow compressors are shown in Fig. 5.12. These curves are quite similar to the centrifugal compressor. However, axial flow compressors cover a narrower range of mass flow than the centrifugal compressors, and the surge line is also steeper than that of a centrifugal compressor. Surging and choking limit the curves at the two ends. However, the surge points in the axial flow compressors are reached before the curves reach a maximum value. In practice, the design points is very close to the surge line. Therefore, the operating range of axial flow compressors is quite narrow.

Illustrative Example 5.1: In an axial flow compressor air enters the compressor at stagnation pressure and temperature of 1 bar and 292K, respectively. The pressure ratio of the compressor is 9.5. If isentropic efficiency of the compressor is 0.85, find the work of compression and the final temperature at the outlet. Assume $\gamma = 1.4$, and $C_p = 1.005$ kJ/kg K.

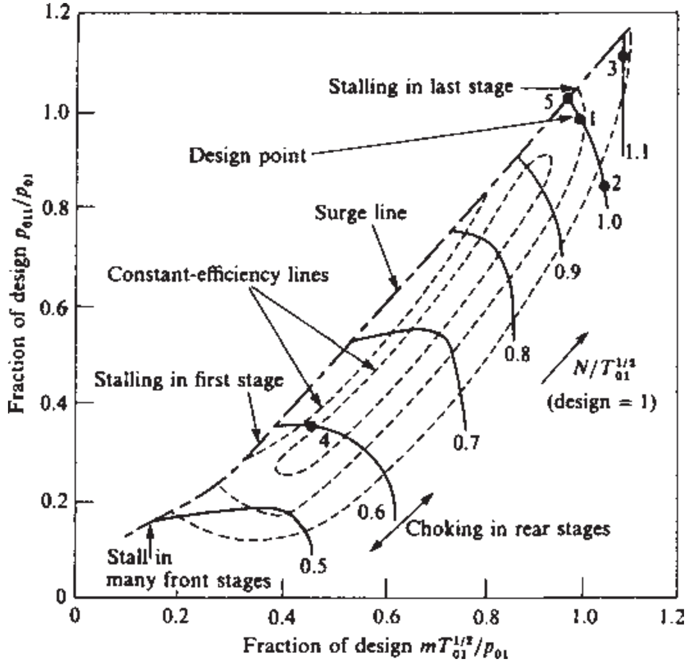

Figure 5.12 Axial flow compressor characteristics.

Solution:

$T_{01} = 292\text{K}, \quad P_{01} = 1\,\text{bar}, \quad \eta_c = 0.85.$

Using the isentropic $P-T$ relation for compression processes,

$$\frac{P_{02}}{P_{01}} = \left[\frac{T'_{02}}{T_{01}}\right]^{\frac{\gamma}{\gamma-1}}$$

where $T_{02'}$ is the isentropic temperature at the outlet.

Therefore,

$$T'_{02} = T_{01}\left[\frac{P_{02}}{P_{01}}\right]^{\frac{\gamma-1}{\gamma}} = 292(9.5)^{0.286} = 555.92\,\text{K}$$

Now, using isentropic efficiency of the compressor in order to find the actual temperature at the outlet,

$$T_{02} = T_{01} + \frac{\left(T'_{02} - T_{01}\right)}{\eta_c} = 292 + \frac{(555.92 - 292)}{0.85} = 602.49\,\text{K}$$

Work of compression:

$$W_c = C_p(T_{02} - T_{01}) = 1.005(602.49 - 292) = 312\,\text{kJ/kg}$$

Illustrative Example 5.2: In one stage of an axial flow compressor, the pressure ratio is to be 1.22 and the air inlet stagnation temperature is 288K. If the stagnation temperature rise of the stages is 21K, the rotor tip speed is 200 m/s, and the rotor rotates at 4500 rpm, calculate the stage efficiency and diameter of the rotor.

Solution:

The stage pressure ratio is given by:

$$R_s = \left[1 + \frac{\eta_s \Delta T_{0s}}{T_{01}}\right]^{\frac{\gamma}{\gamma-1}}$$

or

$$1.22 = \left[1 + \frac{\eta_s(21)}{288}\right]^{3.5}$$

that is,

$$\eta_s = 0.8026 \quad \text{or} \quad 80.26\%$$

The rotor speed is given by:

$$U = \frac{\pi DN}{60}, \quad \text{or} \quad D = \frac{(60)(200)}{\pi(4500)} = 0.85\ \text{m}$$

Illustrative Example 5.3: An axial flow compressor has a tip diameter of 0.95 m and a hub diameter of 0.85 m. The absolute velocity of air makes an angle of 28° measured from the axial direction and relative velocity angle is 56°. The absolute velocity outlet angle is 56° and the relative velocity outlet angle is 28°. The rotor rotates at 5000 rpm and the density of air is 1.2 kg/m^3. Determine:

1. The axial velocity.
2. The mass flow rate.
3. The power required.
4. The flow angles at the hub.
5. The degree of reaction at the hub.

Solution:

1. Rotor speed is given by:

$$U = \frac{\pi DN}{60} = \frac{\pi(0.95)(5000)}{60} = 249\,\text{m/s}$$

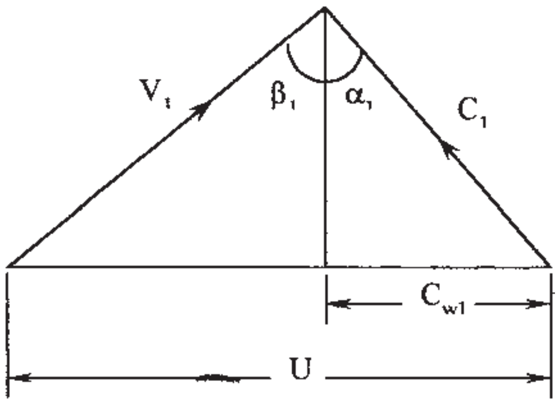

Figure 5.13 Inlet velocity triangle.

Blade speed at the hub:

$$U_h = \frac{\pi D_h N}{60} = \frac{\pi(0.85)(5000)}{60} = 223\,\text{m/s}$$

From the inlet velocity triangle (Fig. 5.13),

$$\tan\alpha_1 = \frac{C_{w1}}{C_a} \qquad \text{and} \qquad \tan\beta_1 = \frac{(U - C_{w1})}{C_a}$$

Adding the above two equations:

$$\frac{U}{C_a} = \tan\alpha_1 + \tan\beta_1$$

or:

$$U = C_a(\tan 28^\circ + \tan 56^\circ) = C_a(2.0146)$$

Therefore, $C_a = 123.6$ m/s (constant at all radii)

2. The mass flow rate:

$$\dot{m} = \pi(r_t^2 - r_h^2)\rho C_a$$
$$= \pi(0.475^2 - 0.425^2)(1.2)(123.6) = 20.98\,\text{kg/s}$$

3. The power required per unit kg for compression is:

$$W_c = \lambda U C_a(\tan\beta_1 - \tan\beta_2)$$
$$= (1)(249)(123.6)(\tan 56^\circ - \tan 28^\circ)10^{-3}$$
$$= (249)(123.6)(1.483 - 0.53)$$
$$= 29.268\,\text{kJ/kg}$$

The total power required to drive the compressor is:

$$W_c = \dot{m}(29.268) = (20.98)(29.268) = 614\,\text{kW}$$

4. At the inlet to the rotor tip:

$$C_{w1} = C_a \tan\alpha_1 = 123.6 \tan 28^\circ = 65.72\,\text{m/s}$$

Using free vortex condition, i.e., $C_w r$ = constant, and using h as the subscript for the hub,

$$C_{w1h} = C_{w1t}\frac{r_t}{r_h} = (65.72)\frac{0.475}{0.425} = 73.452\,\text{m/s}$$

At the outlet to the rotor tip,

$$C_{w2t} = C_a \tan\alpha_2 = 123.6 \tan 56^\circ = 183.24\,\text{m/s}$$

Therefore,

$$C_{w2h} = C_{w2t}\frac{r_t}{r_h} = (183.24)\frac{0.475}{0.425} = 204.8\,\text{m/s}$$

Hence the flow angles at the hub:

$$\tan\alpha_1 = \frac{C_{w1h}}{C_a} = \frac{73.452}{123.6} = 0.594 \text{ or, } \alpha_1 = 30.72^\circ$$

$$\tan\beta_1 = \frac{(U_h)}{C_a} - \tan\alpha_1 = \frac{223}{123.6} - 0.5942 = 1.21$$

i.e., $\beta_1 = 50.43^\circ$

$$\tan\alpha_2 = \frac{C_{w2h}}{C_a} = \frac{204.8}{123.6} = 1.657$$

i.e., $\alpha_2 = 58.89^\circ$

$$\tan\beta_2 = \frac{(U_h)}{C_a} - \tan\alpha_2 = \frac{223}{123.6} - \tan 58.59^\circ = 0.1472$$

i.e., $\beta_2 = 8.37^\circ$

5. The degree of reaction at the hub is given by:

$$\Lambda_h = \frac{C_a}{2U_h}(\tan\beta_1 + \tan\beta_2) = \frac{123.6}{(2)(223)}(\tan 50.43^\circ + \tan 8.37^\circ)$$

$$= \frac{123.6}{(2)(223)}(1.21 + 0.147) = 37.61\%$$

Illustrative Example 5.4: An axial flow compressor has the following data:

Blade velocity at root:	140 m/s
Blade velocity at mean radius:	185 m/s
Blade velocity at tip:	240 m/s
Stagnation temperature rise in this stage:	15K
Axial velocity (constant from root to tip):	140 m/s
Work done factor:	0.85
Degree of reaction at mean radius:	50%

Calculate the stage air angles at the root, mean, and tip for a free vortex design.

Solution:

Calculation at mean radius:

From Eq. (5.1), $W_c = U(C_{w2} - C_{w1}) = U\Delta C_w$

or:

$$C_p(T_{02} - T_{01}) = C_p\Delta T_{0s} = \lambda U\Delta C_w$$

So:

$$\Delta C_w = \frac{C_p\Delta T_{0s}}{\lambda U} = \frac{(1005)(15)}{(0.85)(185)} = 95.87\,\text{m/s}$$

Since the degree of reaction (Fig. 5.14) at the mean radius is 50%, $\alpha_1 = \beta_2$ and $\alpha_2 = \beta_1$.

From the velocity triangle at the mean,

$$U = \Delta C_w + 2C_{w1}$$

or:

$$C_{w1} = \frac{U - \Delta C_w}{2} = \frac{185 - 95.87}{2} = 44.57\,\text{m/s}$$

Hence,

$$\tan\alpha_1 = \frac{C_{w1}}{C_a} = \frac{44.57}{140} = 0.3184$$

that is,

$$\alpha_1 = 17.66° = \beta_2$$

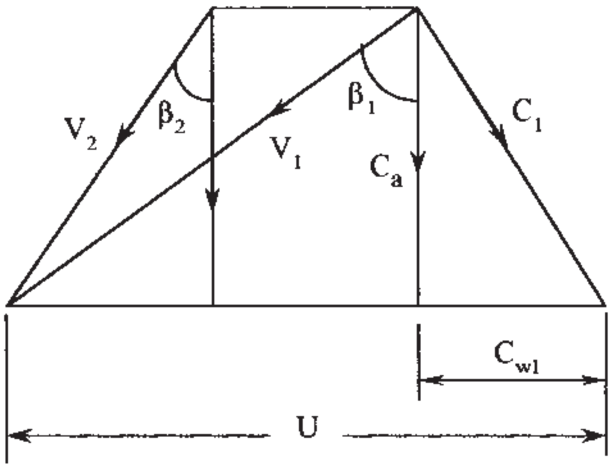

Figure 5.14 Velocity triangle at the mean radius.

and

$$\tan\beta_1 = \frac{(\Delta C_w + C_{w1})}{C_a} = \frac{(95.87 + 44.57)}{140} = 1.003$$

i.e., $\beta_1 = 45.09° = \alpha_2$

Calculation at the blade tip:
Using the free vortex diagram (Fig. 5.15),

$$(\Delta C_w \times U)_t = (\Delta C_w \times U)_m$$

Therefore,

$$\Delta C_w = \frac{(95.87)(185)}{240} = 73.9\,\text{m/s}$$

Whirl velocity component at the tip:

$$C_{w1} \times 240 = (44.57)(185)$$

Therefore:

$$C_{w1} = \frac{(44.57)(185)}{240} = 34.36\,\text{m/s}$$

$$\tan\alpha_1 = \frac{C_{w1}}{C_a} = \frac{34.36}{140} = 0.245$$

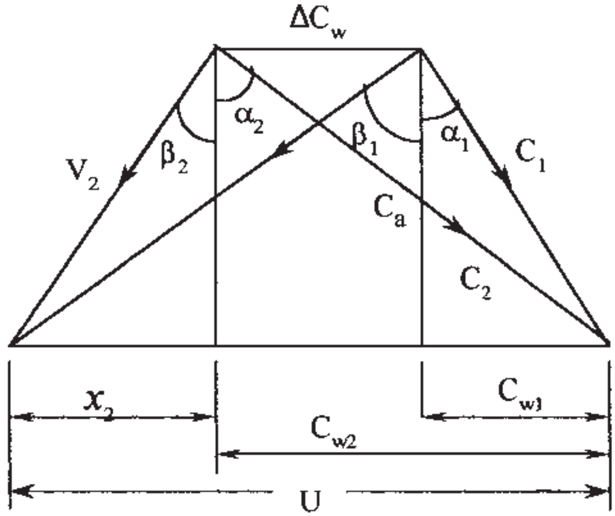

Figure 5.15 Velocity triangles at tip.

Therefore,

$$\alpha_1 = 13.79°$$

From the velocity triangle at the tip,

$$x_2 + \Delta C_w + C_{w1} = U$$

or:

$$x_2 = U - \Delta C_w - C_{w1} = 240 - 73.9 - 34.36 = 131.74$$

$$\tan \beta_1 = \frac{\Delta C_w + x_2}{C_a} = \frac{73.9 + 131.74}{140} = 1.469$$

i.e., $\beta_1 = 55.75°$

$$\tan \alpha_2 = \frac{(C_{w1} + \Delta C_w)}{C_a} = \frac{(34.36 + 73.9)}{140} = 0.7733$$

i.e., $\alpha_2 = 37.71°$

$$\tan \beta_2 = \frac{x_2}{C_a} = \frac{131.74}{140} = 0.941$$

i.e., $\beta_2 = 43.26°$

Calculation at the blade root:

$$(\Delta C_w \times U)_r = (\Delta C_w \times U)_m$$

or:

$$\Delta C_w \times 140 = (95.87)(185) \quad \text{and} \quad \Delta C_w = 126.69\,\text{m/s}$$

Also:

$$(C_{w1} \times U)_r = (C_{w1} \times U)_m$$

or:

$$C_{w1} \times 140 = (44.57)(185) \quad \text{and} \quad C_{w1} = 58.9\,\text{m/s}$$

and

$$(C_{w2} \times U)_t = (C_{w2} \times U)_r$$

so:

$$C_{w2,tip} = C_a \tan\alpha_2 = 140 \tan 37.71^{\circ} = 108.24\,\text{m/s}$$

Therefore:

$$C_{w2,root} = \frac{(108.24)(240)}{140} = 185.55\,\text{m/s}$$

$$\tan\alpha_1 = \frac{58.9}{140} = 0.421$$

i.e., $\alpha_1 = 22.82°$

From the velocity triangle at the blade root, (Fig. 5.16)

or:

$$x_2 = C_{w2} - U = 185.55 - 140 = 45.55$$

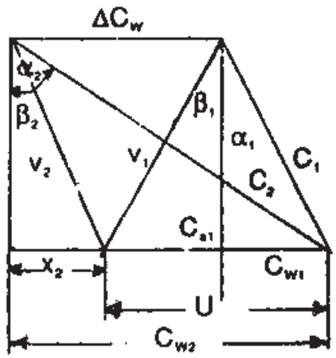

Figure 5.16 Velocity triangles at root.

Therefore:

$$\tan \beta_1 = \frac{U - C_{w1}}{C_a} = \frac{140 - 58.9}{140} = 0.579$$

i.e., $\beta_1 = 30.08°$

$$\tan \alpha_2 = \frac{C_{w2}}{C_a} = \frac{185.55}{140} = 1.325$$

i.e., $\alpha_2 = 52.96°$

$$\tan \beta_2 = -\frac{x_2}{C_a} = -\frac{45.55}{140} = -0.325$$

i.e., $\beta_2 = -18°$

Design Example 5.5: From the data given in the previous problem, calculate the degree of reaction at the blade root and tip.

Solution:

Reaction at the blade root:

$$\Lambda_{root} = \frac{C_a}{2U_r}(\tan \beta_{1r} + \tan \beta_{2r}) = \frac{140}{(2)(140)}(\tan 30.08^{\circ} + \tan(-18^{\circ}))$$

$$= \frac{140}{(2)(140)}(0.579 - 0.325) = 0.127, \quad \text{or} \quad 12.7\%$$

Reaction at the blade tip:

$$\Lambda_{tip} = \frac{C_a}{2U_t}(\tan \beta_{1t} + \tan \beta_{2t}) = \frac{140}{(2)(240)}(\tan 55.75^{\circ} + \tan 43.26^{\circ})$$

$$= \frac{140}{(2)(240)}(1.469 + 0.941) = 0.7029, \quad \text{or} \quad 70.29\%$$

Illustrative Example 5.6: An axial flow compressor stage has the following data:

Air inlet stagnation temperature:	295K
Blade angle at outlet measured from the axial direction:	32°
Flow coefficient:	0.56
Relative inlet Mach number:	0.78
Degree of reaction:	0.5

Find the stagnation temperature rise in the first stage of the compressor.

Solution:

Since the degree of reaction is 50%, the velocity triangle is symmetric as shown in Fig. 5.17. Using the degree of reaction equation [Eq. (5.12)]:

$$\Lambda = \frac{C_a}{2U}(\tan\beta_1 + \tan\beta_2) \quad \text{and} \quad \varphi = \frac{C_a}{U} = 0.56$$

Therefore:

$$\tan\beta_1 = \frac{2\Lambda}{0.56} - \tan 32^\circ = 1.16$$

i.e., $\beta_1 = 49.24^\circ$

Now, for the relative Mach number at the inlet:

$$M_{r1} = \frac{V_1}{(\gamma R T_1)^{\frac{1}{2}}}$$

or:

$$V_1^2 = \gamma R M_{r1}^2 \left(T_{01} - \frac{C_1^2}{2C_p} \right)$$

From the velocity triangle,

$$V_1 = \frac{C_a}{\cos\beta_1}, \quad \text{and} \quad C_1 = \frac{C_a}{\cos\alpha_1}$$

and:

$$\alpha_1 = \beta_2 (\text{since } \Lambda = 0.5)$$

Therefore:

$$C_1 = \frac{C_a}{\cos 32^\circ} = \frac{C_a}{0.848}$$

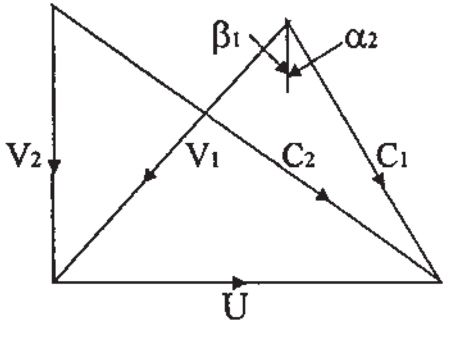

Figure 5.17 Combined velocity triangles for Example 5.6.

and:

$$V_1 = \frac{C_a}{\cos 49.24°} = \frac{C_a}{0.653}$$

Hence:

$$C_1^2 = \frac{C_a^2}{0.719}, \quad \text{and} \quad V_1^2 = \frac{C_a^2}{0.426}$$

Substituting for V_1 and C_1,

$$C_a^2 = 104.41\left(295 - \frac{C_a^2}{1445}\right), \quad \text{so :} \quad C_a = 169.51 \text{ m/s}$$

The stagnation temperature rise may be calculated as:

$$T_{02} - T_{01} = \frac{C_a^2}{C_p \varphi}(\tan \beta_1 - \tan \beta_2)$$

$$= \frac{169.51^2}{(1005)(0.56)}(\tan 49.24° - \tan 32°) = 27.31\text{K}$$

Design Example 5.7: An axial flow compressor has the following design data:

Inlet stagnation temperature:	290K
Inlet stagnation pressure:	1 bar
Stage stagnation temperature rise:	24K
Mass flow of air:	22kg/s
Axialvelocity through the stage:	155.5m/s
Rotational speed:	152rev/s
Work done factor:	0.93
Mean blade speed:	205m/s
Reaction at the mean radius:	50%

Determine: (1) the blade and air angles at the mean radius, (2) the mean radius, and (3) the blade height.

Solution:

(1) The following equation provides the relationship between the temperature rise and the desired angles:

$$T_{02} - T_{01} = \frac{\lambda U C_a}{C_p}(\tan\beta_1 - \tan\beta_2)$$

or:

$$24 = \frac{(0.93)(205)(155.5)}{1005}(\tan\beta_1 - \tan\beta_2)$$

so:

$$\tan\beta_1 - \tan\beta_2 = 0.814$$

Using the degree of reaction equation:

$$\Lambda = \frac{C_a}{2U}(\tan\beta_1 + \tan\beta_2)$$

Hence:

$$\tan\beta_1 + \tan\beta_2 = \frac{(0.5)(2)(205)}{155.5} = 1.318$$

Solving the above two equations simultaneously for β_1 and β_2,

$$2\tan\beta_1 = 2.132,$$

$$\text{so}: \beta_1 = 46.83° = \alpha_2 \text{ (since the degree of reaction is 50\%)}$$

and:

$$\tan\beta_2 = 1.318 - \tan 46.83° = 1.318 - 1.066,$$

$$\text{so}: \beta_2 = 14.14° = \alpha_1$$

(2) The mean radius, r_m, is given by:

$$r_m = \frac{U}{2\pi N} = \frac{205}{(2\pi)(152)} = 0.215\text{m}$$

(3) The blade height, h, is given by:
$m = \rho A C_{a,}$ where A is the annular area of the flow.

$$C_1 = \frac{C_a}{\cos\alpha_1} = \frac{155.5}{\cos 14.14°} = 160.31 \text{ m/s}$$

$$T_1 = T_{01} - \frac{C_1^2}{2C_p} = 290 - \frac{160.31^2}{(2)(1005)} = 277.21 \text{ K}$$

Using the isentropic $P-T$ relation:

$$\frac{P_1}{P_{01}} = \left(\frac{T_1}{T_{01}}\right)^{\frac{\gamma}{\gamma-1}}$$

Static pressure:

$$P_1 = (1)\left(\frac{277.21}{290}\right)^{3.5} = 0.854\,\text{bar}$$

Then:

$$\rho_1 = \frac{P_1}{RT_1} = \frac{(0.854)(100)}{(0.287)(277.21)} = 1.073\,\text{kg/m}^3$$

From the continuity equation:

$$A = \frac{22}{(1.073)(155.5)} = 0.132\text{m}^2$$

and the blade height:

$$h = \frac{A}{2\pi r_m} = \frac{0.132}{(2\pi)(0.215)} = 0.098\text{m}$$

Illustrative Example 5.8: An axial flow compressor has an overall pressure ratio of 4.5:1, and a mean blade speed of 245 m/s. Each stage is of 50% reaction and the relative air angles are the same (30°) for each stage. The axial velocity is 158 m/s and is constant through the stage. If the polytropic efficiency is 87%, calculate the number of stages required. Assume $T_{01} = 290\text{K}$.

Solution:

Since the degree of reaction at the mean radius is 50%, $\alpha_1 = \beta_2$ and $\alpha_2 = \beta_1$. From the velocity triangles, the relative outlet velocity component in the x-direction is given by:

$$V_{x2} = C_a \tan\beta_2 = 158 \tan 30° = 91.22\,\text{m/s}$$

$$V_1 = C_2 = \left[(U - V_{x2})^2 + C_a^2\right]^{\frac{1}{2}}$$

$$= \left[(245 - 91.22)^2 + 158^2\right]^{\frac{1}{2}} = 220.48\,\text{m/s}$$

$$\cos\beta_1 = \frac{C_a}{V_1} = \frac{158}{220.48} = 0.7166$$

so: $\beta_1 = 44.23°$

Stagnation temperature rise in the stage,

$$\Delta T_{0s} = \frac{UC_a}{C_p}(\tan\beta_1 - \tan\beta_2)$$

$$= \frac{(245)(158)}{1005}(\tan 44.23° - \tan 30°) = 15.21\text{K}$$

Number of stages

$$R = \left[1 + \frac{N\Delta T_{0s}}{T_{01}}\right]^{\frac{n}{n-1}}$$

$$\frac{n}{n-1} = \eta_\infty \frac{\gamma}{\gamma - 1} = 0.87\frac{1.4}{0.4} = 3.05$$

Substituting:

$$4.5 = \left[1 + \frac{N15.21}{290}\right]^{3.05}$$

Therefore,

$$N = 12 \text{ stages.}$$

Design Example 5.9: In an axial flow compressor, air enters at a stagnation temperature of 290K and 1 bar. The axial velocity of air is 180 m/s (constant throughout the stage), the absolute velocity at the inlet is 185 m/s, the work done factor is 0.86, and the degree of reaction is 50%. If the stage efficiency is 0.86, calculate the air angles at the rotor inlet and outlet and the static temperature at the inlet of the first stage and stage pressure ratio. Assume a rotor speed of 200 m/s.

Solution:

For 50% degree of reaction at the mean radius (Fig. 5.18), $\alpha_1 = \beta_2$ and $\alpha_2 = \beta_1$.

From the inlet velocity triangle,

$$\cos\alpha_1 = \frac{C_a}{C_1} = \frac{180}{185} = 0.973$$

i.e., $\alpha_1 = 13.35° = \beta_2$

From the same velocity triangle,

$$C_{w1} = \left(C_1^2 - C_a^2\right)^{\frac{1}{2}} = \left(185^2 - 180^2\right)^{\frac{1}{2}} = 42.72\text{ m/s}$$

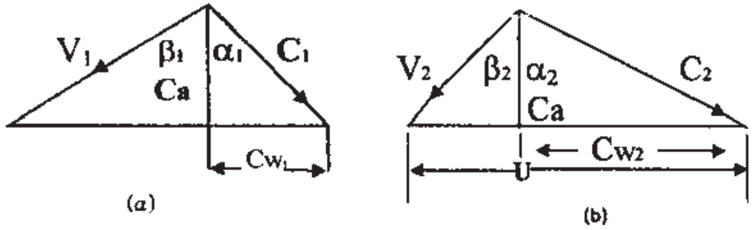

Figure 5.18 Velocity triangles (a) inlet, (b) outlet.

Therefore,

$$\tan\beta_1 = \frac{(U - C_{w1})}{C_a} = \frac{(200 - 42.72)}{180} = 0.874$$

i.e., $\beta_1 = 41.15° = \alpha_2$

Static temperature at stage inlet may be determined by using stagnation and static temperature relationship as given below:

$$T_1 = T_{01} - \frac{C_1}{2C_p} = 290 - \frac{185^2}{2(1005)} = 273\,\mathrm{K}$$

Stagnation temperature rise of the stage is given by

$$\Delta T_{0s} = \frac{\lambda U C_a}{C_p}\left(\tan\beta_1 - \tan\beta_2\right)$$

$$= \frac{0.86(200)(180)}{1005}(0.874 - 0.237) = 19.62\,\mathrm{K}$$

Stage pressure ratio is given by

$$R_s = \left[1 + \frac{\eta_s \Delta T_{0s}}{T_{01}}\right]^{\gamma/\gamma-1} = \left[1 + \frac{0.86 \times 19.62}{290}\right]^{3.5} = 1.22$$

Illustrative Example 5.10: Find the isentropic efficiency of an axial flow compressor from the following data:

Pressure ratio:	6
Polytropic efficiency:	0.85
Inlet temperature:	285 K

Solution:

Using the isentropic P–T relation for the compression process,

$$T_{02'} = T_{01}\left(\frac{P_{02}}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}} = 285(6)^{0.286} = 475.77\,\text{K}$$

Using the polytropic P–T relation for the compression process:

$$\frac{n-1}{n} = \frac{\gamma-1}{\gamma\eta_{\infty,c}} = \frac{0.4}{1.4(0.85)} = 0.336$$

Actual temperature rise:

$$T_{02} = T_{01}\left(\frac{p_{02}}{p_{01}}\right)^{(n-1)/n} = 285(6)^{0.336} = 520.36\,\text{K}$$

The compressor isentropic efficiency is given by:

$$\eta_c = \frac{T_{02'} - T_{01}}{T_{02} - T_{01}} = \frac{475.77 - 285}{520 - 285} = 0.8105, \quad \text{or} \quad 81.05\%$$

Design Example 5.11: In an axial flow compressor air enters the compressor at 1 bar and 290K. The first stage of the compressor is designed on free vortex principles, with no inlet guide vanes. The rotational speed is 5500 rpm and stagnation temperature rise is 22K. The hub tip ratio is 0.5, the work done factor is 0.92, and the isentropic efficiency of the stage is 0.90. Assuming an inlet velocity of 145 m/s, calculate

1. The tip radius and corresponding rotor air angles, if the Mach number relative to the tip is limited to 0.96.
2. The mass flow at compressor inlet.
3. The stagnation pressure ratio and power required to drive the compressor.
4. The rotor air angles at the root section.

Solution:

(1) As no inlet guide vanes

$$\alpha_1 = 0, C_{w1} = 0$$

Stagnation temperature, T_{01}, is given by

$$T_{01} = T_1 + \frac{C_1}{2C_{p2}}$$

or

$$T_1 = T_{01} - \frac{C_1}{2C_p} = 290 - \frac{145^2}{2 \times 1005} = 288.9\text{K}$$

The Mach number relative to tip is

$$M = \frac{V_1}{\sqrt{\gamma R T_1}}$$

or

$$V_1 = 0.96(1.4 \times 287 \times 288.9)^{0.5} = 340.7\ \text{m/s}$$

i.e., relative velocity at tip = 340.7 m/s

From velocity triangle at inlet (Fig. 5.3)

$$V_1^2 = U_t^2 + C_1^2 \text{ or } U_t = \left(340.7^2 - 145^2\right)^{0.5} = 308.3\ \text{m/s}$$

or tip speed,

$$U_t = \frac{2\pi r_t N}{60}$$

or

$$r_t = \frac{308.3 \times 60}{2\pi \times 5500} = 0.535\text{m}.$$

$$\tan\beta_1 = \frac{U_t}{C_a} = \frac{308.3}{145} = 2.126$$

$$\text{i.e., } \beta_1 = 64.81^\circ$$

Stagnation temperature rise

$$\Delta T_{0s} = \frac{\tau U C_a}{C_p}\left(\tan\beta_1 - \tan\beta_2\right)$$

Substituting the values, we get

$$22 = \frac{0.92 \times 308.3 \times 145}{1005}\left(\tan\beta_1 - \tan\beta_2\right)$$

or

$$\left(\tan\beta_1 - \tan\beta_2\right) = 0.538$$

(2) Therefore, $\tan\beta_2 = 1.588$ and $\beta_2 = 57.8^\circ$

$$\text{root radius/tip radius} = \frac{r_m - h/2}{r_m + h/2} = 0.5$$

(where subscript m for mean and h for height)
or $r_m - h/2 = 0.5\ r_m + 0.25\ h$
$\therefore r_m = 1.5\ h$
but $r_t = r_m + h/2 = 1.5\ h + h/2$

or $0.535 = 2\,h$ or $h = 0.268$ m
and $r_m = 1.5\,h = 0.402$ m
Area, $A = 2\pi r_m h = 2\pi \times 0.402 \times 0.268 = 0.677\ \text{m}^2$
Now, using isentropic relationship for p–T

$$\frac{p_1}{p_{01}} = \left(\frac{T_1}{T_{01}}\right)^{\gamma/(\gamma-1)} \quad \text{or} \quad p_1 = 1 \times \left(\frac{288.9}{290}\right)^{3.5} = 0.987\ \text{bar}$$

and

$$\rho_1 = \frac{p_1}{RT_1} = \frac{0.987 \times 10^5}{287 \times 288.9} = 1.19\ \text{kg/m}^3$$

Therefore, the mass flow entering the stage

$$\dot{m} = \rho A C_a = 1.19 \times 0.677 \times 145 = 116.8\ \text{kg/s}$$

(3) Stage pressure ratio is

$$R_s = \left[1 + \frac{\eta_s \Delta T_{0s}}{T_{01}}\right]^{\gamma/(\gamma-1)}$$

$$= \left[1 + \frac{0.90 \times 22}{290}\right]^{3.5} = 1.26$$

Now,

$$W = C_p \Delta T_{0s} = 1005 \times 22 = 22110\ \text{J/kg}$$

Power required by the compressor

$$P = \dot{m} W = 116.8 \times 22110 = 2582.4\ \text{kW}$$

(4) In order to find out rotor air angles at the root section, radius at the root can be found as given below.

$$r_r = r_m - h/2$$

$$= 0.402 - 0.268/2 = 0.267\text{m}.$$

Impeller speed at root is

$$U_r = \frac{2\pi r_r N}{60}$$

$$= \frac{2 \times \pi \times 0.267 \times 5500}{60} = 153.843\ \text{m/s}$$

Therefore, from velocity triangle at root section

$$\tan\beta_1 = \frac{U_r}{C_a} = \frac{153.843}{145} = 1.061$$

i.e., $\beta_1 = 46.9^\circ$

For β_2 at the root section

$$\Delta T_{0s} = \frac{\tau U_r C_a}{C_p}\left(\tan\beta_1 - \tan\beta_2\right)$$

or

$$22 = \frac{0.92 \times 153.843 \times 145}{1005}\left(\tan\beta_1 - \tan\beta_2\right)$$

or

$$\left(\tan\beta_1 - \tan\beta_2\right) = 1.078$$

$$\therefore \beta_2 = -0.974^\circ$$

Design Example 5.12: The following design data apply to an axial flow compressor:

Overall pressure ratio:	4.5
Mass flow:	3.5kg/s
Polytropic efficiency:	0.87
Stagnation temperature rise per stage:	22k
Absolute velocity approaching the last rotor:	160m/s
Absolute velocity angle, measured from the axial direction:	20°
Work done factor:	0.85
Mean diameter of the last stage rotor is:	18.5cm
Ambient pressure:	1.0bar
Ambient temperature:	290K

Calculate the number of stages required, pressure ratio of the first and last stages, rotational speed, and the length of the last stage rotor blade at inlet to the stage. Assume equal temperature rise in all stages, and symmetrical velocity diagram.

Solution:

If N is the number of stages, then overall pressure rise is:

$$R = \left[1 + \frac{N\Delta T_{0s}}{T_{01}}\right]^{\frac{n-1}{n}}$$

where

$$\frac{n-1}{n} = \eta_{\alpha c}\frac{\gamma}{\gamma - 1}$$

(where $\eta_{\alpha c}$ is the polytropic efficiency)
substituting values

$$\frac{n-1}{n} = 0.87 \times \frac{1.4}{0.4} = 3.05$$

overall pressure ratio, R is

$$R = \left[1 + \frac{N \times 22}{290}\right]^{3.05}$$

or

$$(4.5)^{\frac{1}{3.05}} = \left[1 + \frac{N \times 22}{290}\right]$$

$$\therefore \quad N = 8.4$$

Hence number of stages = 8

Stagnation temperature rise, ΔT_{0s}, per stage = 22K, as we took 8 stages, therefore

$$\Delta T_{0s} = \frac{22 \times 8.4}{8} = 23.1$$

From velocity triangle

$$\cos a_8 = \frac{C_{a8}}{C_8}$$

or

$$C_{a8} = 160 \times \cos 20 = 150.35\,\text{m/s}$$

Using degree of reaction, $\Lambda = 0.5$

Then,

$$0.5 = \frac{C_{a8}}{2U}\left(\tan\beta_8 + \tan\beta_9\right)$$

or

$$0.5 = \frac{150.35}{2U}\left(\tan\beta_8 + \tan\beta_9\right) \qquad \text{(A)}$$

Also,

$$\Delta T_{0s} = \frac{\tau U C_{a8}}{C_p}\left(\tan\beta_8 - \tan\beta_9\right)$$

Now, $\Delta T_{0s} = 22K$ for one stage.

As we took 8 stages, therefore;

$$\Delta T_{0s} = \frac{22 \times 8.4}{8} = 23.1\,\text{K}$$

$$\therefore 23.1 = \frac{0.85 \times \text{U} \times 150.35}{1005}\left(\tan\beta_8 - \tan 20\right) \qquad \text{(B)}$$

Because of symmetry, $\alpha_8 = \beta_9 = 20°$

From Eq. (A)

$$U = 150.35\left(\tan\beta_8 + 0.364\right) \qquad \text{(C)}$$

From Eq. (B)

$$U = \frac{181.66}{\tan\beta_8 - 0.364} \qquad \text{(D)}$$

Comparing Eqs. (C) and (D), we have

$$150.35\left(\tan\beta_8 + 0.364\right) = \frac{181.66}{\left(\tan\beta_8 - 0.364\right)}$$

or

$$\left(\tan^2\beta_8 - 0.364^2\right) = \frac{181.66}{150.35} = 1.21$$

or

$$\tan^2\beta_8 = 1.21 + 0.1325 = 1.342$$

$$\therefore \tan\beta_8 = \sqrt{1.342} = 1.159$$

i.e., $\beta_8 = 49.20^{\circ}$

Substituting in Eq. (C)

$$U = 150.35(\tan 49.20° + 0.364)$$
$$= 228.9\,\text{m/s}$$

The rotational speed is given by

$$N = \frac{228.9}{2\pi \times 0.0925} = 393.69\text{rps}$$

In order to find the length of the last stage rotor blade at inlet to the stage, it is necessary to calculate stagnation temperature and pressure ratio of the last stage.
Stagnation temperature of last stage: Fig. 5.19

$$\begin{aligned} T_{o8} &= T_{01} + 7 \times T_{0s} \\ &= 290 + 7 \times 23.1 = 451.7\,\text{K} \end{aligned}$$

Pressure ratio of the first stage is:

$$R = \left[\frac{1 + 1 \times 23.1}{451.7}\right]^{3.05}$$

Now,

$$p_{08}/p_{09} = 1.1643$$

$$\frac{p_{09}}{p_{01}} = 4, \quad \text{and} \quad p_{09} = 4\text{bar}$$

$$p_{08} = \frac{4}{1.1643} = 3.44\text{bar}$$

and

$$T_{08} = T_8 + \frac{C_8^2}{2C_p}$$

or

$$\begin{aligned} T_8 &= T_{08} - \frac{C_8^2}{2C_p} \\ &= 451.7 - \frac{160^2}{2 \times 1005} \\ &= 438.96\,\text{K} \end{aligned}$$

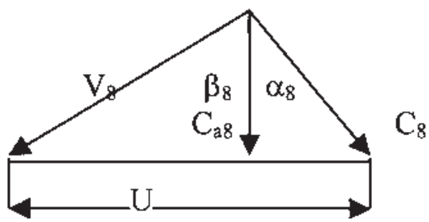

Figure 5.19 Velocity diagram of last stage.

Using stagnation and static isentropic temperature relationship for the last stage, we have

$$\frac{p_8}{p_{08}} = \left(\frac{T_8}{T_{08}}\right)^{1.4/0.4}$$

Therefore,

$$p_8 = 3.44\left(\frac{438.96}{451.7}\right)^{3.5} = 3.112\text{bar}$$

and

$$\rho_8 = \frac{p_8}{RT_8} = \frac{3.112 \times 10^5}{287 \times 438.9} = 2.471\ \text{kg/m}^3$$

Using mass flow rate

$$\dot{m} = \rho_8 A_8 C_{a8}$$

or

$$3.5 = 2.471 \times A_8 \times 150.35$$

$$\therefore A_8 = 0.0094m^2 = 2\pi rh$$

or

$$h = \frac{0.0094}{2\pi \times 0.0925} = 0.0162m$$

i.e., length of the last stage rotor blade at inlet to the stage, $h = 16.17$ mm.

Design Example 5.13: A 10-stage axial flow compressor is designed for stagnation pressure ratio of 4.5:1. The overall isentropic efficiency of the compressor is 88% and stagnation temperature at inlet is 290K. Assume equal temperature rise in all stages, and work done factor is 0.87. Determine the air angles of a stage at the design radius where the blade speed is 218 m/s. Assume a constant axial velocity of 165 m/s, and the degree of reaction is 76%.

Solution:

No. of stages = 10

The overall stagnation temperature rise is:

$$T_0 = \frac{T_{01}\left(R^{\frac{\gamma-1}{\gamma}} - 1\right)}{\eta_c} = \frac{290\left(4.5^{0.286} - 1\right)}{0.88} = 155.879\ \text{K}$$

The stagnation temperature rise of a stage

$$T_{0s} = \frac{155.879}{10} = 15.588\,\mathrm{K}$$

The stagnation temperature rise in terms of air angles is:

$$T_{0s} = \frac{\tau U C_a}{C_p}(\tan\alpha_2 - \tan\alpha_1)$$

or

$$(\tan\alpha_2 - \tan\alpha_1) = \frac{T_{0s} \times C_p}{\tau U C_a} = \frac{15.588 \times 1005}{0.87 \times 218 \times 165} = 0.501 \quad \text{(A)}$$

From degree of reaction

$$\wedge = \left[1 - \frac{C_a}{2U}(\tan\alpha_2 + \tan\alpha_1)\right]$$

or

$$0.76 = \left[1 - \frac{165}{2 \times 218}(\tan\alpha_2 + \tan\alpha_1)\right]$$

$$\therefore (\tan\alpha_2 + \tan\alpha_1) = \frac{0.24 \times 2 \times 218}{165} = 0.634 \quad \text{(B)}$$

Adding (A) and (B), we get

$2\tan\alpha_2 = 1.135$

or $\tan\alpha_2 = 0.5675$

i.e., $\alpha_2 = 29.57°$

and $\tan\alpha_1 = 0.634 - 0.5675 = 0.0665$

i.e., $\alpha_1 = 3.80°$

Similarly, for β_1 and β_2, degree of reaction

$\tan\beta_1 + \tan\beta_2 = 2.01$

and $\tan\beta_1 - \tan\beta_2 = 0.501$

$\therefore 2\tan\beta_1 = 2.511$

i.e., $\beta_1 = 51.46°$

and $\tan\beta_2 = 1.1256 - 0.501 = 0.755$

i.e., $\beta_2 = 37.03°$

Design Example 5.14: An axial flow compressor has a tip diameter of 0.9 m, hub diameter of 0.42 m, work done factor is 0.93, and runs at 5400 rpm. Angles of absolute velocities at inlet and exit are 28 and 58°, respectively and velocity diagram is symmetrical. Assume air density of 1.5 kg/m^3, calculate mass flow rate, work absorbed by the compressor, flow angles and degree of reaction at the hub for a free vortex design.

Solution:

Impeller speed is

$$U = \frac{2\pi r N}{60} = \frac{2\pi \times 0.45 \times 5400}{60} = 254.57 \text{ m/s}$$

From velocity triangle

$$U = C_a(\tan\alpha_1 + \tan\beta_1)$$

$$C_a = \frac{U}{\tan\alpha_1 + \tan\beta_1} = \frac{254.57}{(\tan 28° + \tan 58°)} = 119.47 \text{ m/s}$$

Flow area is

$$A = \pi[r_{tip} - r_{root}]$$
$$= \pi[0.45^2 - 0.42^2] = 0.0833 \text{ m}^2$$

Mass flow rate is

$$\dot{m} = \rho A C_a = 1.5 \times 0.0833 \times 119.47 = 14.928 \text{ kg/s}$$

Power absorbed by the compressor

$$= \tau U(C_{w2} - C_{w1})$$
$$= \tau U C_a(\tan\alpha_2 - \tan\alpha_1)$$
$$= 0.93 \times 254.57 \times 119.47(\tan 58° - \tan 28°)$$
$$= 30213.7 \text{ Nm}$$

$$\text{Total Power, } P = \frac{\dot{m} \times 30213.7}{1000} \text{ kW}$$
$$= 451 \text{ kW}$$

and whirl velocity at impeller tip $C_{wt} = C_a \tan\alpha_1 = 119.47 \times \tan 28° = 63.52$ m/s

Now using free vortex condition
$r\,C_w$ = constant
$\therefore r_h C_{w1h} = r_t C_{w1t}$ (where subscripts h for hub and t for tip)

or

$$C_{w1h} = \frac{r_t C_{w1t}}{r_h} = \frac{0.45 \times 63.52}{0.4} = 71.46\ \text{m/s}$$

Similarly

$$C_{w2t} = C_a \tan\alpha_2 = 119.47 \tan 58° = 191.2\ \text{m/s}$$

and

$$r_h C_{w2h} = r_t C_{w2t}$$

or

$$C_{w2h} = \frac{r_t C_{w2t}}{r_h} = \frac{0.45 \times 191.2}{0.4} = 215.09\ \text{m/s}$$

Therefore, the flow angles at the hub are

$$\tan\alpha_1 = \frac{C_{w1h}}{C_a} \quad \text{(where } C_a \text{ is constant)}$$

$$= \frac{71.46}{119.47} = 0.598$$

i.e., $\alpha_1 = 30.88°$

$$\tan\beta_1 = \frac{U_h - C_a \tan\alpha_1}{C_a}$$

where U_h at the hub is given by

$$U_h = 2\pi r_h N = \frac{2 \times \pi \times 0.4 \times 5400}{60} = 226.29\ \text{m/s}$$

$$\therefore \tan\beta_1 = \frac{226.29 - 119.47 \tan 30.88°}{119.47}$$

i.e., $\beta_1 = 52.34°$

$$\tan\alpha_2 = \frac{C_{w2h}}{C_a} = \frac{215.09}{119.47} = 1.80$$

i.e., $\alpha_2 = 60.95°$
Similarly,

$$\tan\beta_2 = \frac{U_h - C_a \tan\alpha_2}{C_a} = \frac{226.29 - 119.47 \tan 60.95°}{119.47}$$

i.e., $\beta_2 = 5.36°$

Finally, the degree of reaction at the hub is

$$\wedge = \frac{C_a}{2U_h}(\tan\beta_1 + \tan\beta_2) = \frac{119.47}{2 \times 226.29}(\tan 52.34° + \tan 5.36°)$$

$$= 0.367 \text{ or } 36.7\%.$$

Design Example 5.15: An axial flow compressor is to deliver 22 kg of air per second at a speed of 8000 rpm. The stagnation temperature rise of the first stage is 20 K. The axial velocity is constant at 155 m/s, and work done factor is 0.94. The mean blade speed is 200 m/s, and reaction at the mean radius is 50%. The rotor blade aspect ratio is 3, inlet stagnation temperature and pressure are 290 K and 1.0 bar, respectively. Assume C_p for air as 1005 J/kg K and $\gamma = 1.4$. Determine:

1. The blade and air angles at the mean radius.
2. The mean radius.
3. The blade height.
4. The pitch and chord.

Solution:

1. Using Eq. (5.10) at the mean radius

$$T_{02} - T_{01} = \frac{\tau U C_a}{C_p}(\tan\beta_1 - \tan\beta_2)$$

$$20 = \frac{0.94 \times 200 \times 155}{1005}(\tan\beta_1 - \tan\beta_2)$$

$$(\tan\beta_1 - \tan\beta_2) = 0.6898$$

Using Eq. (5.12), the degree of reaction is

$$\wedge = \frac{C_a}{2U}(\tan\beta_1 + \tan\beta_2)$$

or

$$(\tan\beta_1 + \tan\beta_2) = \frac{0.5 \times 2 \times 200}{155} = 1.29$$

Solving above two equations simultaneously

$$2\tan\beta_1 = 1.98$$

$$\therefore \beta_1 = 44.71° = \alpha_2 \text{ (as the diagram is symmetrical)}$$

$$\tan\beta_2 = 1.29 - \tan 44.71°$$

i.e.,

$$\beta_2 = 16.70° = \alpha_1$$

2. Let r_m be the mean radius

$$r_m = \frac{U}{2\pi N} = \frac{200 \times 60}{2\pi \times 8000} = 0.239\text{m}$$

3. Using continuity equation in order to find the annulus area of flow

$$C_1 = \frac{C_a}{\cos\alpha_1} = \frac{155}{\cos 16.70°} = 162\text{ m/s}$$

$$T_1 = T_{01} - \frac{C_1^2}{2C_p} = 290 - \frac{162^2}{2 \times 1005} = 276.94\text{ K}$$

Using isentropic relationship at inlet

$$\frac{p_1}{p_{01}} = \left(\frac{T_1}{T_{01}}\right)^{\frac{\gamma}{\gamma-1}}$$

Static pressure is

$$p_1 = 1.0\left(\frac{276.94}{290}\right)^{3.5} = 0.851\text{bars}$$

Density is

$$\rho_1 = \frac{p_1}{RT_1} = \frac{0.851 \times 100}{0.287 \times 276.94} = 1.07\text{ kg/m}^3$$

From the continuity equation,

$$A = \frac{22}{1.07 \times 155} = 0.133\text{m}^2$$

Blade height is

$$h = \frac{A}{2\pi r_m} = \frac{0.133}{2 \times \pi \times 0.239} = 0.089\text{m.}$$

4. At mean radius, and noting that blades β, an equivalent to cascade, α, nominal air deflection is

$$\varepsilon = \beta_1 - \beta_2$$
$$= 44.71° - 16.70° = 28.01°$$

Using Fig. 5.20 for cascade nominal deflection vs. air outlet angle, the solidity,

$$\frac{s}{c} = 0.5$$

$$\text{Blade aspect ratio} = \frac{\text{span}}{\text{chord}}$$

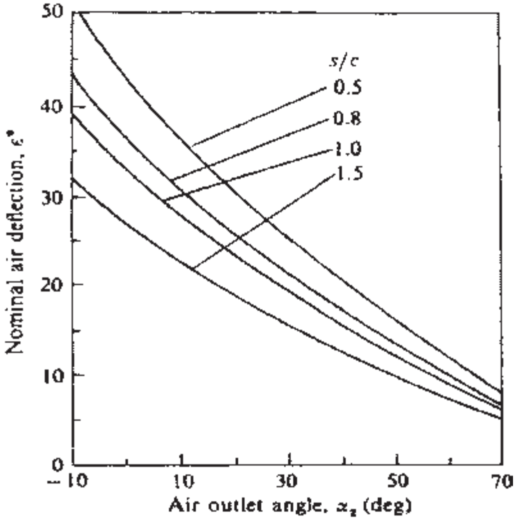

Figure 5.20 Cascade nominal deflection versus air outlet angle.

Blade chord,

$$C = \frac{0.089}{3} = 0.03\text{m}$$

Blade pitch,

$$s = 0.5 \times 0.03 = 0.015\,\text{m}.$$

PROBLEMS

5.1 An axial flow compressor has constant axial velocity throughout the compressor of 152 m/s, a mean blade speed of 162 m/s, and delivers 10.5 kg of air per second at a speed of 10,500 rpm. Each stage is of 50% reaction and the work done factor is 0.92. If the static temperature and pressure at the inlet to the first stage are 288K and 1 bar, respectively, and the stagnation stage temperature rise is 15K, calculate: 1 the mean diameter of the blade row, (2) the blade height, (3) the air exit angle from the rotating blades, and (4) the stagnation pressure ratio of the stage with stage efficiency 0.84.

(0.295 m, 0.062 m, 11.37°, 1.15)

5.2 The following design data apply to an axial flow compressor:

Stagnation temperature rise of the stage:	20 K
Work done factor:	0.90
Blade velocity at root:	155 m/s

Blade velocity at mean radius:	208 m/s
Blade velocity at tip:	255 m/s
Axial velocity (constant through the stage):	155 m/s
Degree of reaction at mean radius:	0.5

Calculate the inlet and outlet air and blade angles at the root, mean radius and tip for a free vortex design.
(18°, 45.5°, 14.84°, 54.07°, 39.71°, 39.18°, 23.56°, 29.42°, 53.75°, −20°)

5.3 Calculate the degree of reaction at the tip and root for the same data as Prob. 5.2.
(66.7%, 10%)

5.4 Calculate the air and blade angles at the root, mean and tip for 50% degree of reaction at all radii for the same data as in Prob. [5.2].
(47.86°, 28.37°, 43.98°, 1.72°)

5.5 Show that for vortex flow,

$$C_w \times r = \text{constant}$$

that is, the whirl velocity component of the flow varies inversely with the radius.

5.6 The inlet and outlet angles of an axial flow compressor rotor are 50 and 15°, respectively. The blades are symmetrical; mean blade speed and axial velocity remain constant throughout the compressor. If the mean blade speed is 200 m/s, work done factor is 0.86, pressure ratio is 4, inlet stagnation temperature is equal to 290 K, and polytropic efficiency of the compressor is 0.85, find the number of stages required.
(8 stages)

5.7 In an axial flow compressor air enters at 1 bar and 15°C. It is compressed through a pressure ratio of four. Find the actual work of compression and temperature at the outlet from the compressor. Take the isentropic efficiency of the compressor to be equal to 0.84
. (167.66 kJ/kg, 454.83 K)

5.8 Determine the number of stages required to drive the compressor for an axial flow compressor having the following data: difference between the tangents of the angles at outlet and inlet, i.e., $\tan\beta_1 - \tan\beta_2 = 0.55$. The isentropic efficiency of the stage is 0.84, the stagnation temperature at the compressor inlet is 288K, stagnation pressure at compressor inlet is 1 bar, the overall stagnation pressure rise is 3.5 bar, and the mass flow rate is 15 kg/s. Assume $C_p = 1.005$ kJ/kg K, $\gamma = 1.4$, $\lambda = 0.86$, $\eta_m = 0.99$
(10 stages, 287.5 kW)

5.9 From the data given below, calculate the power required to drive the compressor and stage air angles for an axial flow compressor.

Stagnation temperature at the inlet:	288 K
Overall pressure ratio:	4
Isentropic efficiency of the compressor:	0.88
Mean blade speed:	170 m/s
Axial velocity:	120 m/s
Degree of reaction:	0.5

(639.4 kW, $\beta_1 = 77.8°$, $\beta_2 = -72.69°$)

5.10 Calculate the number of stages from the data given below for an axial flow compressor:

Air stagnation temperature at the inlet:	288 K
Stage isentropic efficiency:	0.85
Degree of reaction:	0.5
Air angles at rotor inlet:	40°
Air angle at the rotor outlet:	10°
Meanblade speed:	180 m/s
Work done factor:	0.85
Overall pressure ratio:	6

(14 stages)

5.11 Derive the expression for polytropic efficiency of an axial flow compressor in terms of:

(a) n and γ
(b) inlet and exit stagnation temperatures and pressures.

5.12 Sketch the velocity diagrams for an axial flow compressor and derive the expression:

$$\frac{P_{02}}{P_{01}} = \left[1 + \frac{\eta_s \Delta T_{0s}}{T_{01}}\right]^{\frac{\gamma}{\gamma-1}}$$

5.13 Explain the term "degree of reaction". Why is the degree of reaction generally kept at 50%?

5.14 Derive an expression for the degree of reaction and show that for 50% reaction, the blades are symmetrical; i.e., $\alpha_1 = \beta_2$ and $\alpha_2 = \beta_1$.

5.16 What is vortex theory? Derive an expression for vortex flow.

5.17 What is an airfoil? Define, with a sketch, the various terms used in airfoil geometry.

NOTATION

C	absolute velocity
C_L	lift coefficient
C_p	specific heat at constant pressure
D	drag
F_x	tangential force on moving blade
h	blade height, specific enthalpy
L	lift
N	number of stage, rpm
n	number of blades
R	overall pressure ratio, gas constant
R_s	stage pressure ratio
U	tangential velocity
V	relative velocity
α	angle with absolute velocity, measured from the axial direction
α_2^*	nominal air outlet angle
β	angle with relative velocity, measure from the axial direction
ΔT_A	static temperature rise in the rotor
ΔT_B	static temperature rise in the stator
ΔT_{0s}	stagnation temperature rise
ΔT_s	static temperature rise
Δ^*	nominal deviation
ϵ^*	nominal deflection
ϵ_s	stalling deflection
φ	flow coefficient
Λ	degree of reaction
λ	work done factor
ψ	stage loading factor

SUFFIXES

1	inlet to rotor
2	outlet from the rotor
3	inlet to second stage
a	axial, ambient
m	mean
r	radial, root
t	tip
w	whirl